Nonlinear Optimization-Based Power-Voltage Control of Grid-Connected Converter in Weak Grid

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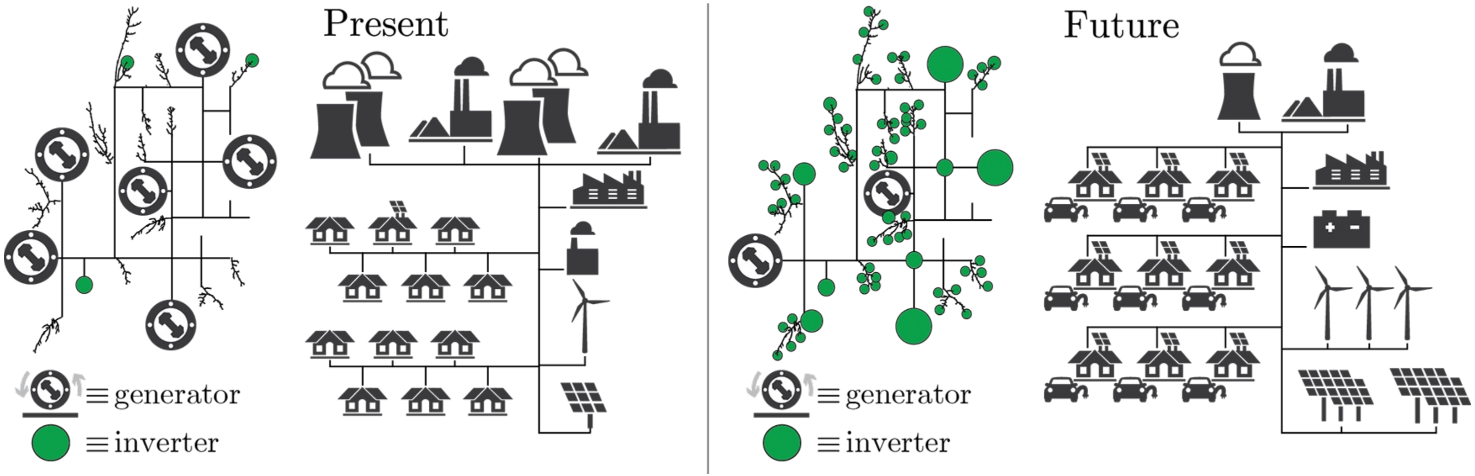
OUTLINE

1. Introduction

Introduction: Grid Becoming Weaker

- Penetration of inverter-based resources (IBRs) into power grid[1]
  - Losing stiff voltage source characteristic = inertia decreasing
  - Low short-circuit ratio (SCR)

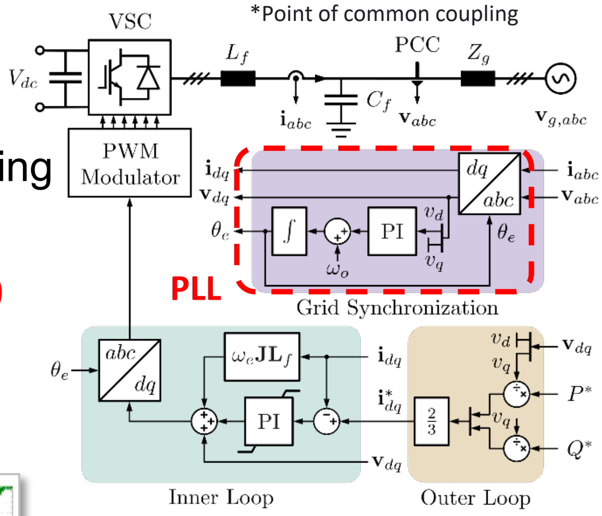


< Existing and future power systems, changing due to penetration of IBRs[2] >

[1] B. Kroposki et al., "Achieving a 100systems with extremely high levels of variable renewable energy," IEEE Power and Energy Magazine, vol. 15, no. 2, pp. 61–73, 2017.
[2] Lin et al., Villegas Pico, Hugo N., Seo, Gab-Su, Pierre, Brian J., and Ellis, Abraham. Research Roadmap on Grid-Forming Inverters. United States: N. p., 2020. Web. doi:10.2172/1721727.

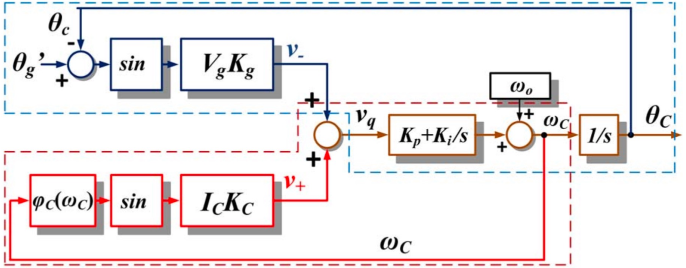
Introduction: Need of Voltage Control

- Grid-following (GFL) control
  - **Outer loop**: generate current reference for power tracking
  - **Inner loop**: current control
  - **Grid synchronization** using **phase-locked loop (PLL)**



< Grid-following control structure >

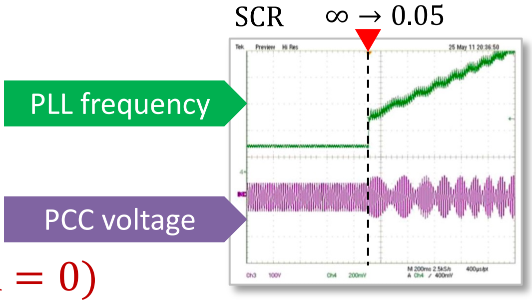
Grid synchronization loop: **stiff grid** $(\mathrm{SCR} = \infty)$



Self synchronization loop: **island mode** $(\mathrm{SCR} = 0)$

➡ **Positive feedback loop**

< Quasi-static PLL model considering grid interaction[3] >



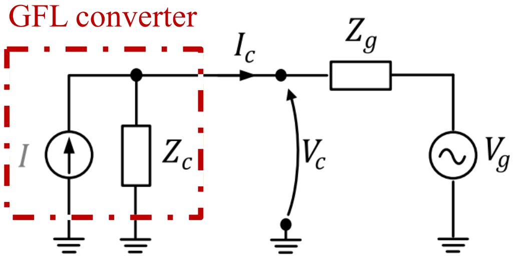
< PLL behavior under inductive local load condition[3] >

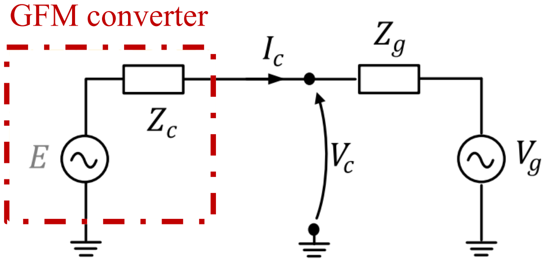
∴ **Voltage control** required under weak grid

[3] D. Dong et al., "Analysis of Phase-Locked Loop Low-Frequency Stability in Three-Phase Grid-Connected Power Converters Considering Impedance Interactions," in IEEE Trans. on Ind. Electron., vol. 62, no. 1, pp. 310-321, Jan. 2015.

Introduction: How about grid-forming (GFM)?

- Control of grid-connected voltage source converter (VSC)[4]



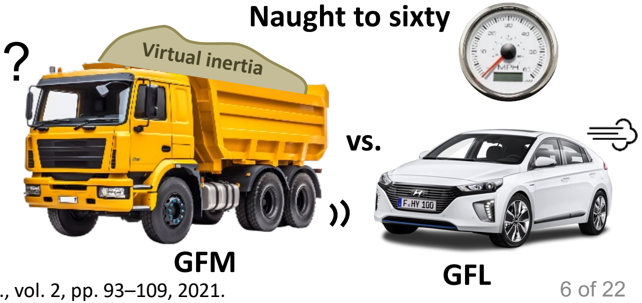


Under weak grid..

Grid Synch.	Mostly use PLL → Unstable	Emulate synchronous generator → Stable
Behavior	Current source	Voltage source
Objective	Accurate power transfer	Regulation of voltage and frequency

- Is GFM control a perfect solution for weak grid?
  - Slow dynamic for emulating inertia
  - Inappropriate for fast power tracking

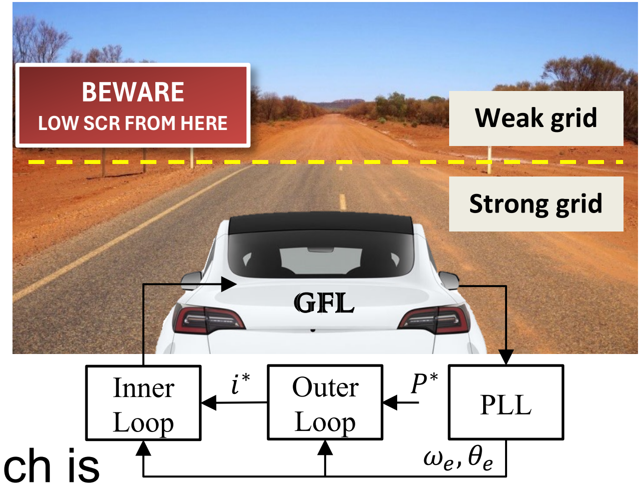
NO!



[4] R. Rosso et al., "Grid- forming converters: Control approaches, grid-synchronization, and future trends—a review," IEEE Open J. of Ind. App., vol. 2, pp. 93–109, 2021.

Introduction: Motivation

- If we want to **keep the fast current source behavior** in **weak grid**…
  - Sol1. Improve negative effect of PLL[5]
    → No rated power injection demonstrated
  - Sol2. Modify overall structure of GFL control[6-7]
    → Complicated tuning/modification of control structure
  - Sol3. Reshape output impedance of VSC[8]
    → PLL BW reduction required for non-unity power factor



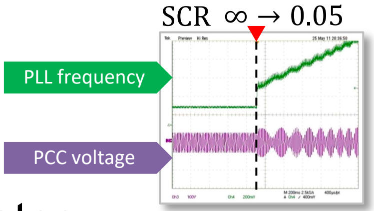
- In this paper,
  Power-voltage control outer loop is suggested which is
  - based on **nonlinear optimization**
  - compatible with inner loop **without modifying structure** or **reducing PLL BW**
  - able to **inject reactive power automatically** and **output rated power**

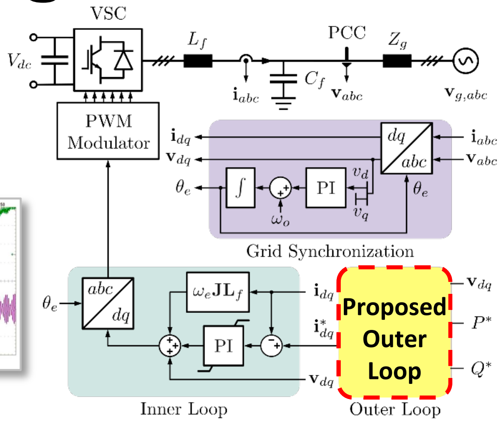
[5] D. Zhu et al., “Improved design of pll controller for lcl-type grid-connected converter in weak grid,” IEEE Trans. on Power Electron., vol. 35, no. 5, pp. 4715–4727, 2020.
[6] C. Li et al., “Tuning method of a grid-following converter for the extremely-weak-grid connection,” IEEE Trans. on Power Sys., vol. 37, no. 4, pp. 3169–3172, 2022.
[7] M. Davari et al., “Robust vector control of a very weak-grid-connected voltage-source converter considering the phase-locked loop dynamics,” IEEE Trans. on Power Electron., vol. 32, no. 2, pp. 977–994, 2017.
[8] M. Li et al., “The control strategy for the grid-connected inverter through impedance reshaping in q-axis and its stability analysis under a weak grid,” IEEE J. of Emerg. and Selec. Topics in Power Electron., vol. 9, no. 3, pp. 3229–3242, 2021.

2. Formulation of Nonlinear Optimization Problem

Basic Idea for Proposed Power-Voltage Control

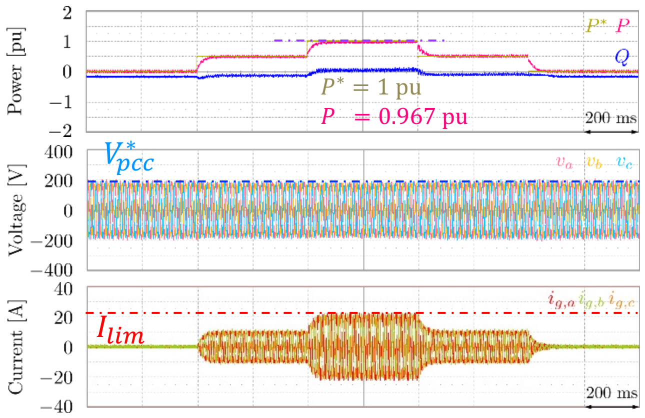
- GFL control under weak grid
  - Necessary for fast power tracking
  - Negative impact of PLL on stability
  - Low SCR = Large grid impedance
    Inject power ➔ PCC voltage fluctuates





< Grid-following control structure >

- **Outer loop** for **power-voltage control**
  - Goal: to generate current reference $\mathbf{i}^*_{dq}$ that
    - minimizes power tracking error
    - controls PCC voltage
    - keeps current below the limit

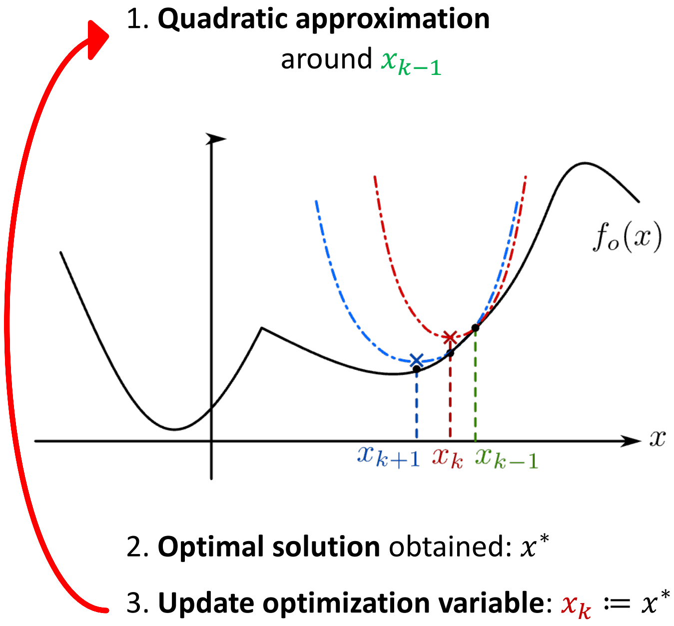


Sequential Quadratic Programming (SQP)

- **Numerical and iterative method** to for nonlinear programming

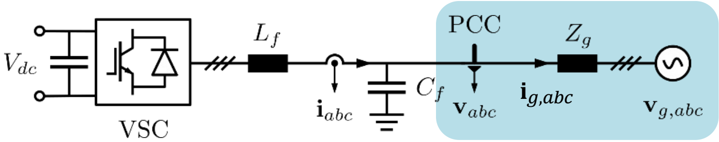
- **Repeat** the sequence below:
  1. Formulate quadratic subproblem
     - Quadratic approximation of objective function
     - Linearization of constraints

     ➔ Convex optimization problem
  2. Obtain optimal solution of subproblem
  3. Update optimization variable



Construction of Nonlinear Optimization Problem

- Goal: to **generate current reference $\mathbf{i}_{dq}^*$** that
  - minimizes **power tracking** error
  - **controls PCC voltage**
  - keeps filter inductor **current below limit**



< 2-level VSC connected to AC grid >

$\mathbf{v}_{dq}$: voltage at PCC
$\mathbf{v}_{g,dq}$: ideal voltage source
$\mathbf{i}_{dq}$: filter inductor current
$\mathbf{i}_{g,dq}$: grid current

Objective function:
Error in power tracking

$$P = \frac{3}{2}\mathbf{v}_{dq}^{\top}\mathbf{i}_{dq} \quad \cdots (2)$$

$$\min_{\mathbf{i}_{dq}} f_o(\mathbf{i}_{dq}) = \frac{1}{2}(P - P^*)^2$$
$$\text{subject to } f_{\mathrm{i}}(\mathbf{i}_{dq}) = \mathbf{i}_{dq}^{\top}\mathbf{i}_{dq} - I_{lim}^2 \le 0$$
$$f_{\mathbf{v}}(\mathbf{i}_{dq}) = \mathbf{v}_{dq}^{\top}\mathbf{v}_{dq} - {V_{pcc}^*}^2 = 0 \quad \cdots (1)$$

Solved using **sequential quadratic programming**[9]

Constraint:
1) Current limitation
2) Voltage control at PCC

$$\mathbf{v}_{dq} = \mathbf{R}_g\mathbf{i}_{g,dq} + \mathbf{L}_g\frac{d\mathbf{i}_{g,dq}}{dt} + \omega_e\mathbf{J}\mathbf{L}_g\mathbf{i}_{g,dq} + \mathbf{v}_{g,dq}$$
$$\mathbf{i}_{dq} = \mathbf{i}_{g,dq} + \mathbf{C}_f\frac{d\mathbf{v}_{dq}}{dt} + \omega_e\mathbf{J}\mathbf{C}_f\mathbf{v}_{dq} \quad \cdots (3)$$

Optimization variable: $\mathbf{i}_{dq}$
Solution: $\mathbf{i}_{dq}^* = \underset{\mathbf{i}_{dq}}{\text{argmin}}\, f_o(\mathbf{i}_{dq})$

[9] P. T. Boggs and J. W. Tolle, "Sequential quadratic programming," *Acta Numerica*, vol. 4, p. 1–51, 1995.

Solving Optimization Problem using SQP

- At k^{th} sampling instant:
  1. Quadratic subproblem of (1)
     - Using Lagrangian (4)
  2. Obtain optimal solution of (5)
     A. Calculate partial derivatives
        $f_o(\cdot), f_{\mathbf{i}}(\cdot), f_{\mathbf{v}}(\cdot)$ are differentiable function of $\mathbf{i}_{dq}$
     B. Apply KKT condition
        *Kuhn-Karush-Tucker (KKT) condition
     C. **Analytic solution** obtained!

Original Nonlinear Optimization Problem (1)

$$\min_{\mathbf{i}_{dq}} f_o(\mathbf{i}_{dq}) = \frac{1}{2}(P - P^*)^2$$
$$\text{subject to } f_{\mathbf{i}}(\mathbf{i}_{dq}) = \mathbf{i}_{dq}^{\top}\mathbf{i}_{dq} - I_{lim}^2 \le 0$$
$$f_{\mathbf{v}}(\mathbf{i}_{dq}) = \mathbf{v}_{dq}^{\top}\mathbf{v}_{dq} - V_{pcc}^* = 0$$

Lagrangian of (1)

$$\mathcal{L}(\mathbf{i}_{dq}, \rho, \nu) = f_o(\mathbf{i}_{dq}) + \rho f_{\mathbf{i}}(\mathbf{i}_{dq}) + \nu f_{\mathbf{v}}(\mathbf{i}_{dq}) \quad \cdots (4)$$

Quadratic approximation around $\mathbf{i}_{dq}[k]$

Convex optimization problem

$$\min_{\Delta\mathbf{i}_{dq}} \left.\frac{\partial f_o}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \Delta\mathbf{i}_{dq} + \frac{1}{2}\Delta\mathbf{i}_{dq}^{\top}\left(\left.\frac{\partial^2 \mathcal{L}}{\partial \mathbf{i}_{dq}^2}\right|_k\right)\Delta\mathbf{i}_{dq}$$

$$\text{subject to } f_{\mathbf{i}}(\mathbf{i}_{dq}[k]) + \left.\frac{\partial f_{\mathbf{i}}}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \Delta\mathbf{i}_{dq} \le 0$$

Constraint 1: **Current limitation**

$$f_{\mathbf{v}}(\mathbf{i}_{dq}[k]) + \left.\frac{\partial f_{\mathbf{v}}}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \Delta\mathbf{i}_{dq} = 0$$

Constraint 2: **Voltage control**

$\cdots (5)$

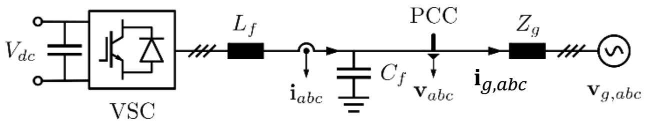
$[k]$: k^{th} sampling instant

$|_k$: partial derivate around $\mathbf{i}_{dq}[k]$

3. Proposed Power-Voltage Control

Analytic Derivation of Optimal Solution

- Case divided based on equality establishment of **current constraint**
  - **Active** case: O.P. = voltage circle ∩ current limit circle (equality)
  - **Inactive** case: O.P. = voltage circle ∩ power tracking line (inequality)



$$\mathbf{v}_{dq} = \mathbf{Z}_g \mathbf{i}_{g,dq} + \mathbf{v}_{g,dq}$$

$^*\mathbf{Z}_g = R_g + L_g \frac{d}{dt} + \omega_e \mathbf{J} L_g$

Quadratic subproblem (5)

$$\min_{\Delta \mathbf{i}_{dq}} \left. \frac{\partial f_o}{\partial \mathbf{i}_{dq}} \right|_k^{\mathsf{T}} \Delta \mathbf{i}_{dq} + \frac{1}{2} \Delta \mathbf{i}_{dq}^{\mathsf{T}} \left(\left. \frac{\partial^2 \mathcal{L}}{\partial \mathbf{i}_{dq}^2} \right|_k \right) \Delta \mathbf{i}_{dq}$$

$$\text{subject to } f_{\mathrm{i}}(\mathbf{i}_{dq}[k]) + \left. \frac{\partial f_{\mathrm{i}}}{\partial \mathbf{i}_{dq}} \right|_k^{\mathsf{T}} \Delta \mathbf{i}_{dq} \leq 0$$

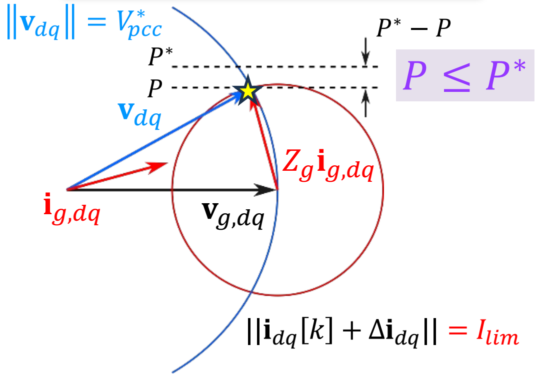
$$f_{\mathrm{v}}(\mathbf{i}_{dq}[k]) + \left. \frac{\partial f_{\mathrm{v}}}{\partial \mathbf{i}_{dq}} \right|_k^{\mathsf{T}} \Delta \mathbf{i}_{dq} = 0$$

Objective:
Minimize power error

Constraint 1:
Current limitation

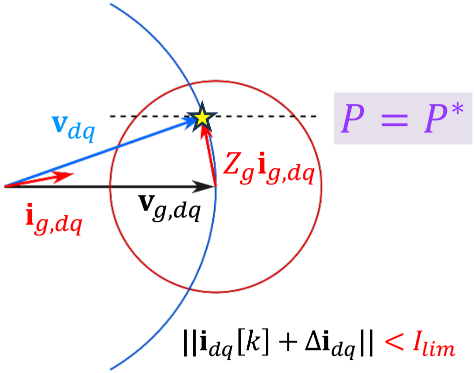
Constraint 2:
Voltage control

Case 1: active solution



- Current **reaches limit**
- Power error may not be zero

Case 2: inactive solution



- Current **below limit**
- Power accurately tracked

★ : operating point (O.P.)

Analytic Derivation of Optimal Solution

- KKT condition applied for each case to derive analytic solution
  - Optimal solution (8) satisfies current limitation
- Generates $\mathbf{i}^*_{dq}[k]$
  - Step size ratio α
    - Related to stability
    - Experimentally set ($\alpha = 0.1$)

Case 1: active solution

$$\min_{\Delta\mathbf{i}_{dq}} \left.\frac{\partial f_o}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \Delta\mathbf{i}_{dq} + \frac{1}{2}\Delta\mathbf{i}_{dq}^{\top}\left(\left.\frac{\partial \mathcal{L}}{\partial \mathbf{i}_{dq}}\right|_k\right)\Delta\mathbf{i}_{dq}$$

$$\text{subject to } f_i(\mathbf{i}_{dq}[k]) + \left.\frac{\partial f_i}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \Delta\mathbf{i}_{dq} = 0$$

$$f_v(\mathbf{i}_{dq}[k]) + \left.\frac{\partial f_v}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \Delta\mathbf{i}_{dq} = 0$$

$$\Delta\mathbf{i}_{dq}^{act} = -\begin{bmatrix} \left.\frac{\partial f_i}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \\ \left.\frac{\partial f_v}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \end{bmatrix}^{-1} \begin{bmatrix} f_i(\mathbf{i}_{dq}[k]) \\ f_v(\mathbf{i}_{dq}[k]) \end{bmatrix} \quad \cdots (6)$$

Case 2: inactive solution

$$\min_{\Delta\mathbf{i}_{dq}} \left.\frac{\partial f_o}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \Delta\mathbf{i}_{dq} + \frac{1}{2}\Delta\mathbf{i}_{dq}^{\top}\left(\left.\frac{\partial \mathcal{L}}{\partial \mathbf{i}_{dq}}\right|_k\right)\Delta\mathbf{i}_{dq}$$

$$\text{subject to } f_i(\mathbf{i}_{dq}[k]) + \left.\frac{\partial f_i}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \Delta\mathbf{i}_{dq} < 0$$

$$f_v(\mathbf{i}_{dq}[k]) + \left.\frac{\partial f_v}{\partial \mathbf{i}_{dq}}\right|_k^{\top} \Delta\mathbf{i}_{dq} = 0$$

$$\begin{bmatrix} \Delta\mathbf{i}_{dq}^{inact} \\ \nu[k] \end{bmatrix} = -\begin{bmatrix} \left.\frac{\partial^2 \mathcal{L}}{\partial \mathbf{i}_{dq}^2}\right|_k & \left.\frac{\partial f_v}{\partial \mathbf{i}_{dq}}\right|_k \\ \left.\frac{\partial f_v}{\partial \mathbf{i}_{dq}}\right|_k^{\top} & 0 \end{bmatrix}^{-1} \begin{bmatrix} \left.\frac{\partial f_o}{\partial \mathbf{i}_{dq}}\right|_k \\ f_v(\mathbf{i}_{dq}[k]) \end{bmatrix} \quad \cdots (7)$$

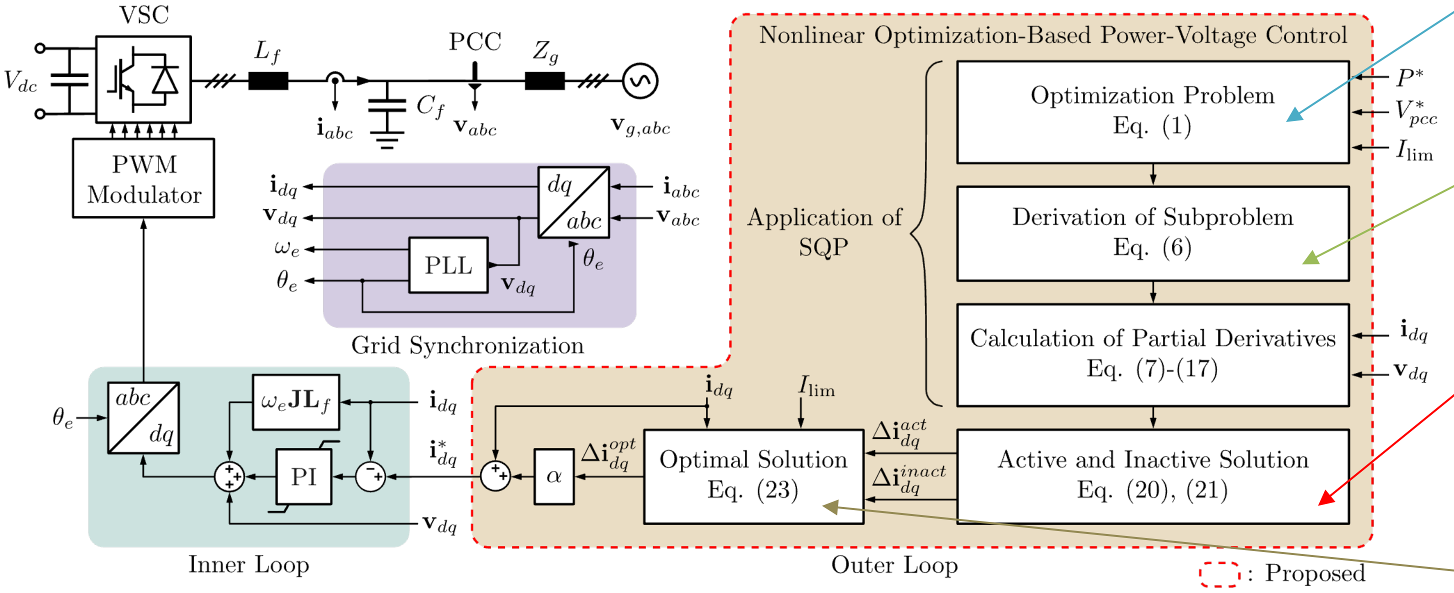
Current reference and optimal solution

$$\mathbf{i}^*_{dq}[k] = \mathbf{i}_{dq}[k] + \alpha\Delta\mathbf{i}_{dq}^{opt} \quad \cdots (8)$$

$$\Delta\mathbf{i}_{dq}^{opt} = \begin{cases} \Delta\mathbf{i}_{dq}^{inact} & ||\mathbf{i}_{dq}[k] + \mathbf{i}_{dq}|| < I_{lim} \\ \Delta\mathbf{i}_{dq}^{act} & otherwise \end{cases}$$

Proposed Power-Voltage Control Outer Loop

- Compatible with PLL and current control inner loop
- No need to modify control structure or tune PLL bandwidth



$$\min_{\mathbf{i}_{dq}} f_o(\mathbf{i}_{dq}) = \frac{1}{2}(P - P^*)^2$$
$$\text{subject to } f_i(\mathbf{i}_{dq}) = \mathbf{i}_{dq}^\top \mathbf{i}_{dq} - I_{lim}^2 \le 0$$
$$f_v(\mathbf{i}_{dq}) = \mathbf{v}_{dq}^\top \mathbf{v}_{dq} - {V_{pcc}^*}^2 = 0$$

$$\min_{\Delta \mathbf{i}_{dq}} \left.\frac{\partial f_o}{\partial \mathbf{i}_{dq}}\right|_k^\top \Delta \mathbf{i}_{dq} + \frac{1}{2}\Delta \mathbf{i}_{dq}^\top \left(\left.\frac{\partial^2 \mathcal{L}}{\partial \mathbf{i}_{dq}^2}\right|_k\right) \Delta \mathbf{i}_{dq}$$
$$\text{subject to } f_i(\mathbf{i}_{dq}[k]) + \left.\frac{\partial f_i}{\partial \mathbf{i}_{dq}}\right|_k^\top \Delta \mathbf{i}_{dq} \le 0$$
$$f_v(\mathbf{i}_{dq}[k]) + \left.\frac{\partial f_v}{\partial \mathbf{i}_{dq}}\right|_k^\top \Delta \mathbf{i}_{dq} = 0$$

$$\Delta \mathbf{i}_{dq}^{act} = -\begin{bmatrix} \left.\frac{\partial f_i}{\partial \mathbf{i}_{dq}}\right|_k^\top \\ \left.\frac{\partial f_v}{\partial \mathbf{i}_{dq}}\right|_k^\top \end{bmatrix}^{-1} \begin{bmatrix} f_i(\mathbf{i}_{dq}[k]) \\ f_v(\mathbf{i}_{dq}[k]) \end{bmatrix}$$

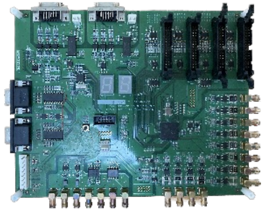
$$\begin{bmatrix} \Delta \mathbf{i}_{dq}^{inact} \\ \nu[k] \end{bmatrix} = -\begin{bmatrix} \left.\frac{\partial^2 \mathcal{L}}{\partial \mathbf{i}_{dq}^2}\right|_k & \left.\frac{\partial f_v}{\partial \mathbf{i}_{dq}}\right|_k \\ \left.\frac{\partial f_v}{\partial \mathbf{i}_{dq}}\right|_k^\top & 0 \end{bmatrix}^{-1} \begin{bmatrix} \left.\frac{\partial f_o}{\partial \mathbf{i}_{dq}}\right|_k \\ f_v(\mathbf{i}_{dq}[k]) \end{bmatrix}$$

$$\Delta \mathbf{i}_{dq}^{opt} = \begin{cases} \Delta \mathbf{i}_{dq}^{inact} & \|\mathbf{i}_{dq}[k] + \mathbf{i}_{dq}\| < I_{lim} \\ \Delta \mathbf{i}_{dq}^{act} & \textit{otherwise} \end{cases}$$

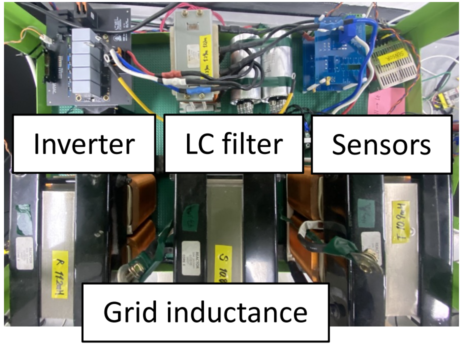
< Control structure of the VSC with the proposed nonlinear optimization-based power-voltage control >

4. Experimental Results

Experimental Setup

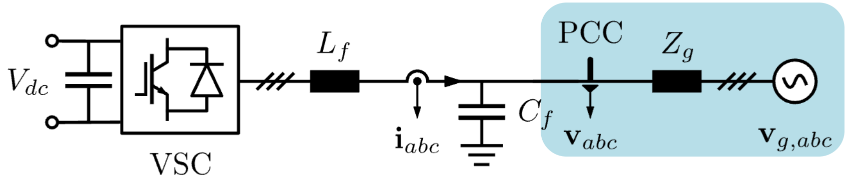


DSP:
TI F28377D



Grid emulator:
California Instruments MX30

Weak grid (SCR=2)



< 2-level VSC connected to AC grid >

< Table 1: Circuit parameters >

Symbol	Parameter	Value	Unit	
V_{dc}	DC-link voltage	450	V	
V_g	Grid line-to-line voltage	220	V	
f_g	Grid line frequency	60	Hz	
S_n	Rated power	5.8	kVA	SCR=2
L_g	Grid inductance	11.1	mH	$(Z_{L_g} = 0.5$ pu)
L_f	Filter inductance	1.2	mH	$(Z_{L_f} = 0.054$ pu)
C_f	Filter capacitance	35	μF	$(G_{C_f} = 0.11$ pu)

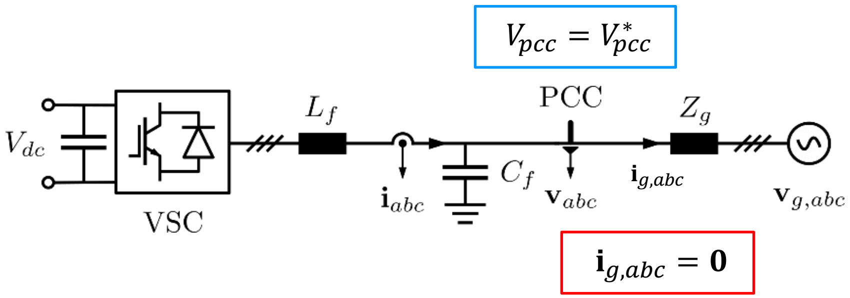
< Table 2: Control parameters >

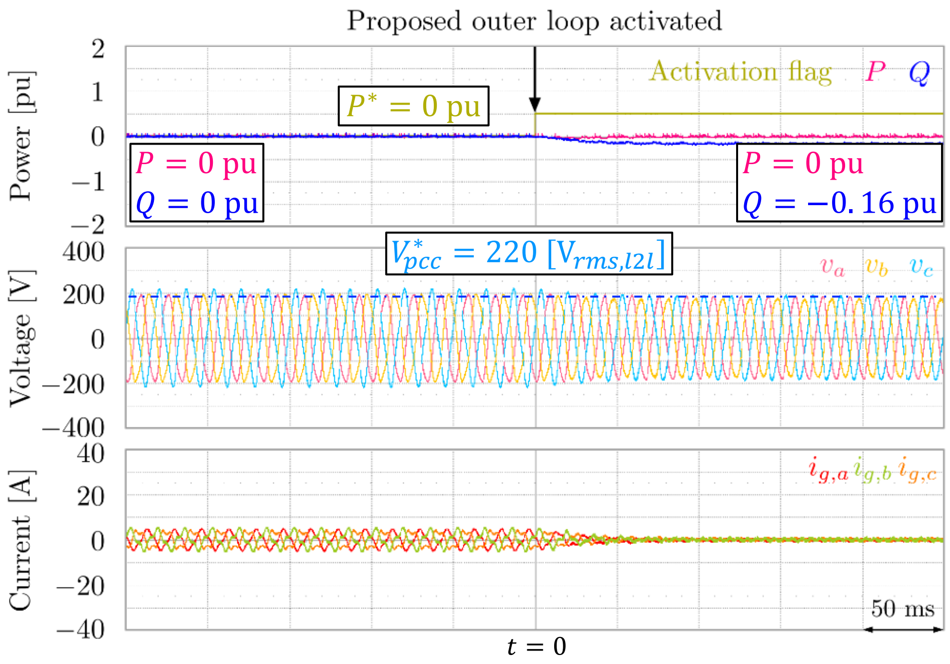
Symbol	Parameter	Value	Unit
f_{sw}	Inverter switching frequency	10	kHz
f_s	Inverter sampling frequency	10	kHz
α	Step size ratio of power-voltage controller	0.1	–
k_{pc}	Proportional gain of current controller	7.54	–
k_{ic}	Integral gain of current controller	502.65	–
k_{pp}	Proportional gain of PLL[10]	0.97	–
k_{ip}	Integral gain of PLL[10]	24.29	–

[10] D. Yang et al., “Symmetri- cal PLL for SISO Impedance Modeling and Enhanced Stability in Weak Grids,” IEEE Transactions on Power Electronics, vol. 35, pp. 1473– 1483, Feb. 2020.

Experimental Result: Voltage Control

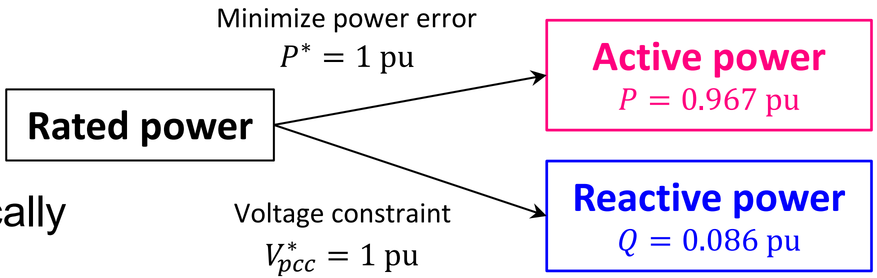
- Outer loop switched from conventional GFL to proposed method at $t = 0$
- Current reference is generated to
  - Control power to be 0 [W]
  - Inject reactive power automatically
  - Control PCC voltage to V_{pcc}^*

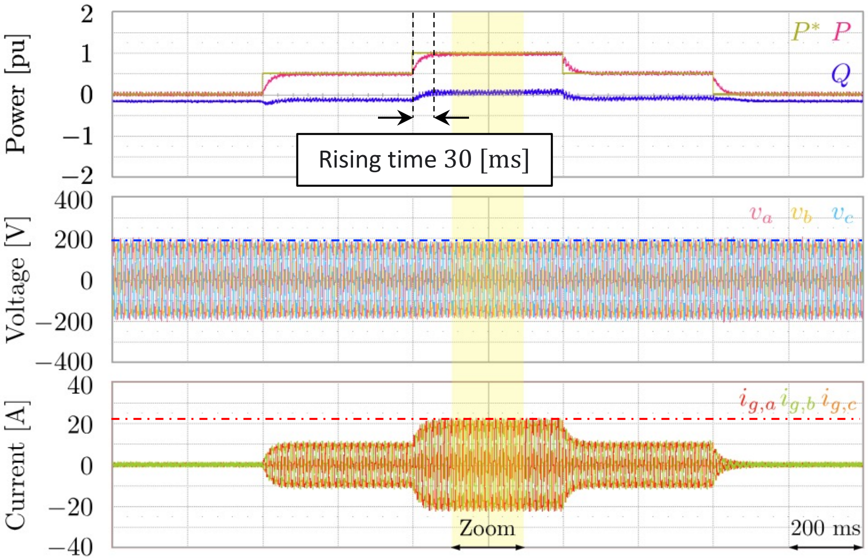


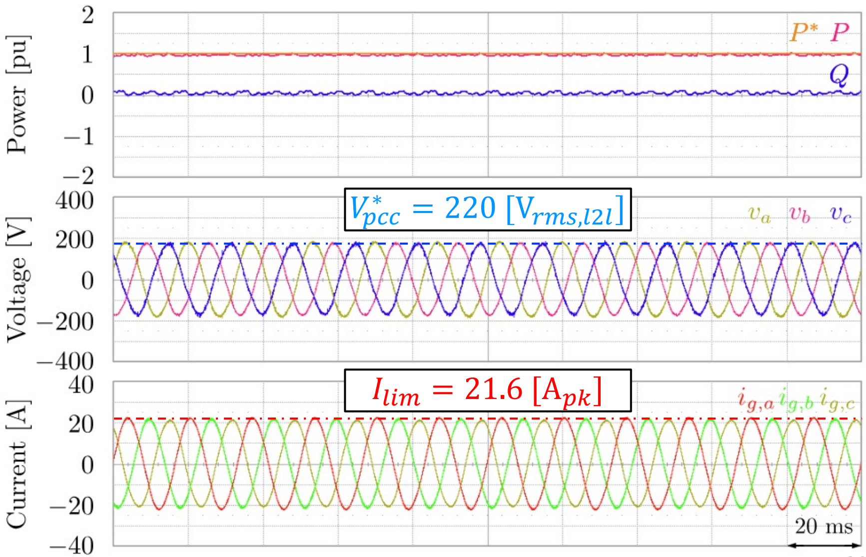


Experimental Result: Power Tracking

- Fast and stable power tracking
- Capacity for rated power divided
  - Into active and reactive power automatically
  - Within current limit of converter I_{lim}







5. Conclusion

Conclusion

- Construction of optimization problem for power-voltage control
  - Minimization of power tracking error
  - Control of voltage at PCC for stable power transfer
  - Generation of optimal current reference considering converter rating

- Proposed power-voltage control outer loop
  - Compatible with conventional current controller and PLL
  - No additional gain tuning of current controller / reduction of PLL bandwidth
  - Real-time implementation available by deriving analytic solution[11]

- Contributions and Limitations

  (+) Fast and stable rated power injection under weak grid of SCR = 2

  (-) Complicated stability analysis

[11] J. Park, H. -J. Cho, H. Kim and S. -K. Sul, "Online Torque Control of IPMSM for Flux Weakening Operation Using Sequential Quadratic Programming," 2023 IEEE Energy Conversion Congress and Exposition (ECCE), Nashville, TN, USA, 2023, pp. 4750-4755.

Thank you

Appendix

Appendix: KKT Condition

- Need to be satisfied to solve an arbitrary optimization problem using Lagrangian function
- Necessary and sufficient condition of convex optimization
- **KKT Conditions**:

$$\min_{\mathbf{x}} f(\mathbf{x})$$
$$\text{subject to } g_i(\mathbf{x}) \leq 0, i = 1, \dots, m$$
$$h_j(\mathbf{x}) = 0, j = 1, \dots, k$$

1. Satisfy primal constraints

$$g_i(\mathbf{x}) \leq 0, h_j(\mathbf{x}) = 0$$

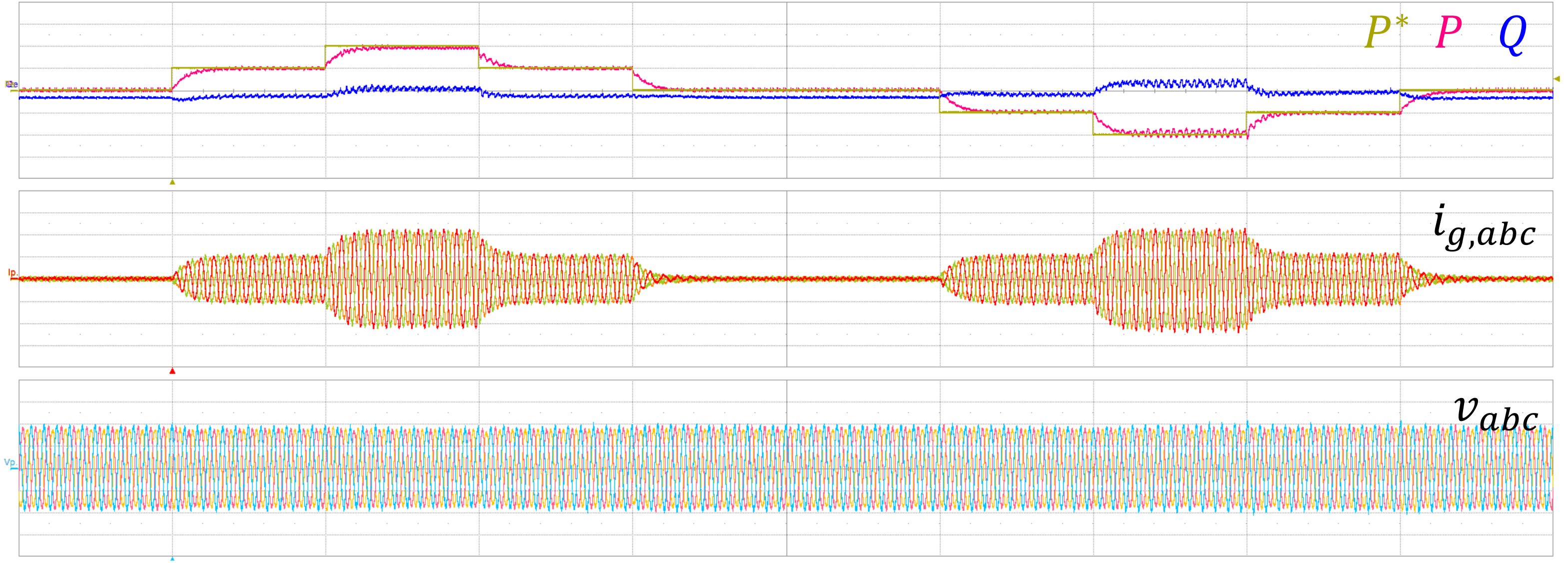
2. Satisfy dual constraints

$$\mathcal{L}(\mathbf{x}, \lambda, \nu) = f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i g_i(\mathbf{x}) + \sum_{j=1}^{k} \nu_j h_j(\mathbf{x}), \qquad \lambda_i \geq 0$$

3. Complementary slackness $\lambda_i g_i(\mathbf{x}^*) = 0$
4. Gradient of Lagrangian is 0 w.r.t opt. variable $\nabla_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \lambda, \nu) = 0$

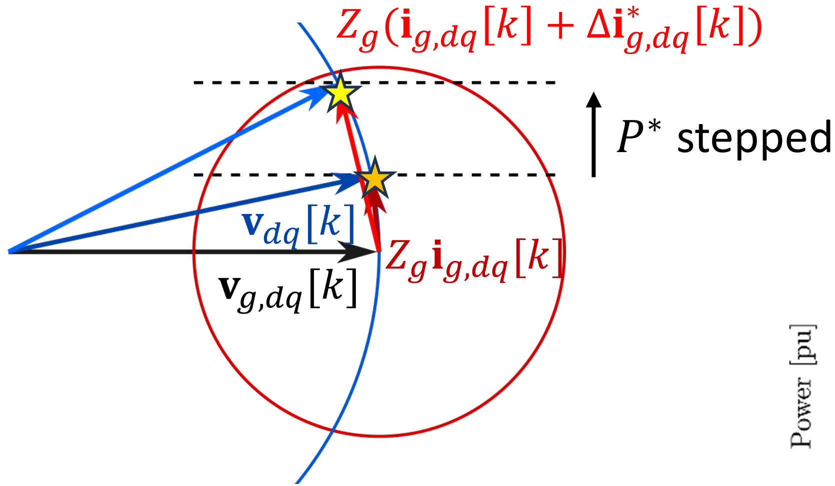
Appendix: Negative Power Injection

- Available to inject negative power up to $-1\ \mathrm{pu}$
  - Larger oscillation in power at steady state



Appendix: Step Size Ratio and Stability

- How much will you move toward the ‘current’ optimal point?
  - Current optimal point ≠ final destination
  - Though, constraints should be satisfied at every point
- With small step size ratio, takes long time to reach new steady state
  - **Equivalent BW of proposed outer loop can be calculated as** $\alpha\omega_{cc}$



Step size	Rising time	Stability
↓	↑	↑

Rising time 30 [ms] ($\alpha = 0.1$)